Institut lecentrale Equations differentielles (Serie proposée) ①

FACULTÉ DES SCIENCES ET TECHNIQUES DE TANGER

DÉPARTEMENT DES SCIENCES MATHÉMATIQUES

MIPC$_I$
Feuille de T.D $N^0 6$

Année Universitaire 06-07
Equations différentielles

Exercice 1: EQUATIONS À VARIABLE SEPARÉS: Résoudre les équations suivantes:

$(1+y)y' = x-1$, $y'e^y = e^x$, $yy'/(1+y^2) = 1/(x^2-1)$,
$-yy'/(1-y^2) = x-1$ avec $y(0)=1$

Exercice 2: Equations homogènes: Résoudre les équations suivantes:

$y' = \frac{y}{x} - 1$, $2y' = \frac{y}{x} + \frac{y^2}{x^2}$, $y' = \frac{y}{x} - 2\sqrt{\frac{y}{x}}$, $y' = \frac{y}{x} + 1/\cos\frac{y}{x}$,
$2y' = \frac{x^2+y^2}{xy}$

Exercice 3: EQUATIONS DIFFÉRENTIELLES LINÉAIRES DU PREMIER ORDRE: Résoudre les équation suivantes;

$y' + 1/x^2 y = e^{1/x}$, $y' - y\tan x = e^x$, $y' - 2x - 1/x^2 y = 1$
$y' - \frac{x^2+1}{x(x^2-1)}y = 2y' - \frac{y}{1-x^2} = 1+x$, $y' + y/\sqrt{1+x^2} = 1 - x/\sqrt{1+x^2}$
$y' - 2xy = -(2x-1)e^x$ avec $y(0)=0$

Exercice 4 : Equation de Bernoulli

1/ $y' - y - xy^2 = 0$

2/ $y = y\frac{\tan x}{3} + \frac{1}{3y^2}$

Exercice 5 : Equation de Ricatti

1/ $(1-x^3)y' + x^2y + y^2 - 2x = 0$
(verifier que $y_0 = x^2$ est solution)

2/ $y' = -xy^2 + \frac{y}{x} - \frac{2}{x^3}$; $y_0 = \frac{1}{x^2}$

Exercice 6 Equations differentielle du 2° ordre à coefficients constants

1/ $y'' - 2y' + y = x^2 + x$

2/ $y'' - 2y' = x^2 + x$

3/ $y'' = x^2 + x$

4/ $y'' - 4y' + 3y = (x+1)e^{2x}$

5/ $y'' - 4y' + 3y = (x+1)e^x$

6/ $y'' - 2y' + 1 = (x+1)e^x$

7/ $y'' - 4y' + 3y = \sin 2x$

8/ $y'' - 4y' + 3y = e^x + \sin x$

9/ $y'' - 4y' + 3y = \operatorname{ch} x$

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Exercice 4

1/ $y' - y - xy^2 = 0$ (F). $\alpha = 2$, On pose $z = y^{1-\alpha} = y^{-1} = \frac{1}{y}$, $z' = -y^{-2} \cdot y'$

$: y^2 \rightarrow \frac{y'}{y^2} - \frac{1}{y} - x = 0$ d'où $-z' - z - x = 0$; $z' + z = -x$ (E)

.(E') : $z' + z = 0$; $z' = -z$; $\frac{z'}{z} = -1$; $\ln|z| = -x + k$, $z_1 = ce^{-x}$

. Soit $z_0 = ce^{-x}$ solution particulière de (E) alors $z_0' + z_0 = -x$

$\Rightarrow c'e^{-x} - ce^{-x} + ce^{-x} = -x \Rightarrow c' = -xe^x \Rightarrow c = \int -xe^x dx = (1-x)e^x +$

d'où $z_0 = 1 - x \Rightarrow z = ce^{-x} + 1 - x \Rightarrow y = \frac{1}{ce^{-x} + 1 - x}$ sol de (F)

2/ $y' = y\frac{\tan x}{3} + \frac{1}{3y^2}$; $\alpha = -2$; On pose $z = y^{1-\alpha} = y^3$; $z' = 3y^2 y'$

$\div \frac{1}{y^2} \Rightarrow y^2 y' = y^3 \frac{\tan x}{3} + \frac{1}{3} \Rightarrow \frac{z'}{3} = z\frac{\tan x}{3} + \frac{1}{3} \Rightarrow$ (E) : $z' = z\tan x + 1$

(E') : $z' = z\tan x$; $\frac{z'}{z} = \frac{\sin x}{\cos x} \Rightarrow \ln|z| = -\ln|\cos x| + k \Rightarrow z_1 = \frac{c}{\cos x}$ sol de E'

. $z_0 = \frac{c}{\cos x}$ sol de E ; $\frac{c'\cos x + \sin x \cdot c}{\cos^2 x} = \frac{c}{\cos x} \cdot \frac{\sin x}{\cos x} + 1 \Rightarrow c' = \cos x \Rightarrow c = \sin x$

$\Rightarrow z_0 = \frac{\sin x}{\cos x} = \tan x \Rightarrow z = \frac{c}{\cos x} + \tan x = \frac{c + \sin x}{\cos x} \Rightarrow y = \sqrt[3]{\frac{c + \sin x}{\cos x}}$

Exercice 1/(F) : $(1 - x^3)y' + x^2 y + y^2 - 2x = 0$

. On pose $z = y - x^2$; $y = z + x^2$, $y' = z' + 2x$; (F) devient : (E) : $(1-x^3)z' + 3x^2 z + z^2 = 0$

(E) est une equation de Bernoulli ($\alpha = 2$) : On pose $u = z^{1-\alpha} = z^{-1}$; $u' = -z^{-2}z'$

On divise (E) par z^2 : $(1-x^3)\frac{z'}{z^2} + 3x^2 \cdot \frac{1}{z} + 1 = 0 \Rightarrow (x^3 - 1)u' + 3x^2 u + 1 = 0$ (G)

(G) est une equation linéaire du 1er ordre :

(G') : $(x^3 - 1)u' + 3x^2 u = 0 \Rightarrow \frac{u'}{u} = -\frac{3x^2}{x^3 - 1} \Rightarrow \ln|u| = -\ln|x^3 - 1| + k \Rightarrow u_1 = \frac{c}{x^3 - 1}$ sol de G'

Pour $u_0 = \frac{c}{x^3 - 1}$ solution particulière de G : alors $u_0' = \frac{c'(x^3 - 1) - 3x^2 c}{(x^3 - 1)^2}$

Alors $(x^3 - 1)\frac{c'(x^3 - 1) - 3x^2 c}{(x^3 - 1)^2} + 3x^2 \cdot \frac{c}{x^3 - 1} + 1 = 0 \Rightarrow c' = -1 \Rightarrow c = -x \Rightarrow u_0 = -\frac{x}{x^3 - 1}$

$\Rightarrow u = u_1 + u_0 = \frac{c - x}{x^3 - 1}$ sol de (G) $\Rightarrow z = \frac{1}{u} = \frac{x^3 - 1}{c - x}$ sol de (F) $\Rightarrow y = \frac{x^3 - 1}{c - x} + x^2$ sol de (E)

2/ (F) : $y' = -xy^2 + \frac{y}{x} - \frac{2}{x^3}$; On pose $y = z + \frac{1}{x^2}$, $y' = z' - \frac{2}{x^3}$, (F) devient (E)

(E) : $xz' + xz^2 + z = 0$ equation de Bernoulli ($\alpha = 2$) : on pose $u = z^{1-\alpha} = z^{-1} = \frac{1}{z}$

$u' = -z^{-2} \cdot z'$. on divise (E) par z^2 et on obtient l'equation (G)

(G) :

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Exercice 1

1/ $(1+y)\,y' = x-1$

$\int (1+y) = \int (x-1)\,dx$; $y + \frac{y^2}{2} = \frac{x^2}{2} - 1 + C_1$; $y^2 + 2y = x^2 - 2x + C_2$

$(y+1)^2 - 1 = x^2 - 2x + C$; $(y+1)^2 = x^2 - 2x + C$; $y = -1 \pm \sqrt{x^2 - 2x + C}$

2/ $y' e^y = e^x$

$\int e^y\,dy = \int e^x\,dx$; $e^y = e^x + C$; $y = \ln(e^x + C)$

3/ $\frac{yy'}{1+y^2} = \frac{1}{x^2-1}$. $\int \frac{y}{1+y^2}\,dy = \int \frac{dx}{x^2-1} = \int \frac{1/2}{x-1} - \frac{1/2}{x+1}\,dx$

$\frac{1}{2}\ln(1+y^2) = \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C_1$; $\ln(1+y^2) = \ln\left|\frac{x-1}{x+1}\right| + 2.C_1$

$1+y^2 = \left|\frac{x-1}{x+1}\right| \cdot e^{2C_1}$; $1+y^2 = K\left(\frac{x-1}{x+1}\right)$; $y = \pm\sqrt{K\left(\frac{x-1}{x+1}\right) - 1}$

4/ $\frac{-yy'}{1-y^2} = x-1$; $y(0) = 1$: $\int \frac{-y}{1-y^2}\,dy = \int (x-1)\,dx$

$\int \frac{1/2}{y-1} + \frac{1/2}{y+1}\,dy = \frac{x^2}{2} - x + C$; $\frac{1}{2}\ln|y-1| + \frac{1}{2}\ln|y+1| = \frac{x^2}{2} - x + C$

$\ln|y^2-1| = x^2 - 2x + 2C$; $|y^2-1| = e^{x^2-2x} \cdot e^{2C} \Rightarrow y^2 - 1 = K e^{x^2-2x}$

$y = \pm\left(1 + K e^{x^2-2x}\right)^{1/2}$

Exercice 2

On pose $u = \frac{y}{x}$ alors $y = xu$ et $y' = u + xu'$

1/ $y' = \frac{y}{x} - 1$;

l'equation devient : $u + xu' = u - 1$.

2/ $2y' = \frac{y}{x} + \frac{y^2}{x^2}$

l'equation devient : $2u + 2xu' = u + u^2$, $2xu' = u^2 - u$, $\frac{2u'}{u^2-u} = \frac{1}{x}$

$\int \frac{2\,du}{u^2-u} = \int \frac{1}{x}\,dx$; $\int \frac{2}{u-1} - \frac{2}{u}\,du = \ln|x| + C$; $2\ln|u-1| - 2\ln|u| = \ln|x| + C$

$\ln\left|\frac{u-1}{u}\right| = \ln\sqrt{|x|} + C$; $\left|\frac{u-1}{u}\right| = \sqrt{|x|}\,e^{2C}$ $\Rightarrow$ $\frac{u-1}{u} = K\sqrt{|x|}$

$u(1 - K\sqrt{|x|}) = 1$; $\Rightarrow u = \frac{1}{1-K\sqrt{|x|}}$; $y = \frac{x}{1-K\sqrt{|x|}}$

3/ $y' = \frac{y}{x} - 2\sqrt{\frac{y}{x}}$

l'equation devient : $u + xu' = u - 2\sqrt{u}$; $-\frac{u'}{2\sqrt{u}} = \frac{1}{x}$; $-\sqrt{u} = \ln|x| + C$

$u = (\ln|x| + C)^2$; $y = x(\ln|x| + C)^2$

4/ $y' = \frac{y}{x} + \frac{1}{\cos\frac{y}{x}}$ l'equation devient : $u + xu' = u + \frac{1}{\cos u}$; $u'\cos u = \frac{1}{x}$

$\sin u = \ln|x| + C$; $u = \text{Arc}\sin(\ln|x| + C)$; $y = x\,\text{Arc}\sin(\ln|x| + C)$.

5/ $2y' = \frac{x^2+y^2}{xy} = \frac{x}{y} + \frac{y}{x}$, l'eq devient $2(u+xu') = u + \frac{1}{u}$; $2xu' = \frac{1}{u} - u$

$\frac{2u}{u^2-1} u' = -\frac{1}{x}$; $\left(\frac{1}{u-1} + \frac{1}{u+1}\right) u' = -\frac{1}{x}$; $\ln|u-1| + \ln|u+1| = -\ln|x| + C$

$\ln|u^2-1| = -\ln|x| + C$; $|u^2-1| = e^{-\ln|x|} \cdot e^C$ $\Rightarrow u^2 - 1 = \frac{K}{x}$

$u = \pm\sqrt{1+\frac{K}{x}}$; $y = \pm x\sqrt{1+\frac{K}{x}}$

Exercice 3

1/ $y' + \frac{1}{x^2} y = e^{1/x}$ (E)

. Resolution de (E'): $y' + \frac{1}{x^2} y = 0$; $\frac{y'}{y} = -\frac{1}{x^2}$; $\ln|y| = \frac{1}{x} + C$; $y_1 = ke^{1/x}$

. Solution particuliere de(E) de la forme $y_0 = ke^{1/x}$

$y_0' = k'e^{1/x} + k(-1/x^2)e^{1/x}$

y_0 verifie (E) alors $y_0' + \frac{1}{x^2} y_0 = e^{1/x}$; $k'e^{1/x} - \frac{k}{x^2} e^{1/x} + \frac{k}{x^2} e^{1/x} = e^{1/x} \Rightarrow k' = 1$

$\Rightarrow k = x$ d'ou $y_0 = xe^{1/x}$ et $y = y_1 + y_0 = ke^{1/x} + xe^{1/x} = (k+x)e^{1/x}$ solutions de(E)

2/ (E): $y' - y\tan x = e^x$

. Resolution de (E'): $y' - y\tan x = 0$; $\frac{y'}{y} = \frac{\sin x}{\cos x}$; $\ln|y| = -\ln|\cos x| + C$; $y_1 = \frac{K}{\cos x}$

. Solution particuliere de(E) : de la forme $y_0 = \frac{k}{\cos x}$ alors $y_0' = \frac{k'\cos x + k\sin x}{\cos^2 x}$

On a $y_0' - y_0\tan x = e^x \Rightarrow \frac{k'\cos x + k\sin x}{\cos^2 x} - \frac{k}{\cos x}\cdot\tan x = e^x$; $k' = e^x\cos x$

$k = \int e^x \cos x\, dx = e^x\sin x - \int e^x \sin x\, dx = e^x\sin x - (e^x\cos x + \int e^x\cos x\, dx)$
$\quad u \quad v'$

$k = e^x(\sin x - \cos x) - k \Rightarrow k = \frac{e^x}{2}(\sin x - \cos x) \Rightarrow y_0 = \frac{e^x}{2}(\tan x - 1)$

d'ou $y = y_1 + y_0 = \frac{K}{\cos x} + \frac{e^x}{2}(\tan x - 1)$ solutions de(E)

3/ $y' - \frac{2x-1}{x^2} y = 1$ (E)

. Resolution de (E'): $y' - \frac{2x-1}{x^2} y = 0$; $\frac{y'}{y} = \frac{2x-1}{x^2} = \frac{2}{x} - \frac{1}{x^2}$

$\ln|y| = 2\ln|x| + \frac{1}{x} + C$; $|y| = e^{2\ln|x| + \frac{1}{x} + C}$; $y_1 = kx^2 e^{1/x}$

. Solution particuliere de E de la forme: $y_0 = kx^2 e^{1/x}$

$y_0' = k'(x^2 e^{1/x}) + k(2xe^{1/x} - e^{1/x})$

On a $y_0' - \frac{2x-1}{x^2} y_0 = 1 \Rightarrow k'x^2 e^{1/x} + 2xke^{1/x} - ke^{1/x} - \frac{2x-1}{x^2}\cdot kx^2 e^{1/x} = 1$

$\Rightarrow k'x^2 e^{1/x} = 1 \Rightarrow k' = \frac{1}{x^2} e^{-1/x} \Rightarrow k = e^{-1/x} \Rightarrow y_0 = x^2$ et $y = y_1 + y_0 = kx^2 e^{1/x} + x^2$ solution de E

4/ $y' - \frac{x^2+1}{x(x^2-1)} y = 2$. Resolution de (E'): $y' - \frac{x^2+1}{x(x^2-1)} y = 0$; $\frac{y'}{y} = \frac{x^2+1}{x(x^2-1)} = -\frac{1}{x} + \frac{1}{x-1} + \frac{1}{x+1}$

$\ln|y| = -\ln|x| + \ln|x-1| + \ln|x+1| + C = \ln\left|\frac{x^2-1}{x}\right| + C \Rightarrow |y| = \left|\frac{x^2-1}{x}\right| \cdot e^C \Rightarrow y_1 = K\frac{(x^2-1)}{x}$

. Solution particuliere de (E) : de la forme $y_0 = k\frac{x^2-1}{x}$, $y_0' = k'\frac{x^2-1}{x} + k\cdot\frac{x^2+1}{x}$

$y_0' - \frac{x^2+1}{x(x^2-1)} y_0 = 2 \Rightarrow k'\frac{x^2-1}{x} + k\cdot\frac{x^2+1}{x^2} - \frac{x^2+1}{x^2} - \frac{x^2+1}{x(x^2-1)}\cdot k\cdot\frac{x^2-1}{x} = 2 \Rightarrow k' = \frac{2x}{x^2-1} \Rightarrow k = \ln|x^2-1| \Rightarrow y_0 = \frac{x^2-1}{x}\ln|x^2-1|$

$\Rightarrow y = y_1 + y_0 = K\frac{x^2-1}{x} + \frac{x^2-1}{x}\ln|x^2-1|$ ✓

Exercice

1/ (E): $y'' - 2y' + y = x^2 + x$

• (E'): $y'' - 2y' + y = 0$; Equation caractéristique : $r^2 - 2r + 1 = 0$

$\Delta = 0$; $r_1 = 1$; $y_1 = (\alpha + \beta x)e^x$ solutions de (E')

• $g(x) = x^2 + x$; $b = 1 \neq 0$ donc une solution particulière de (E) est un polynome du second degré ; $y_0 = ax^2 + bx + c$, $y_0' = 2ax + b$; $y_0'' = 2a$

$y_0'' - 2y_0' + y_0 = x^2 + x$; $2a - 2(2ax + b) + ax^2 + bx + c = x^2 + x \Rightarrow \begin{cases} a = 1 \\ -4a + b = 1 \\ c - 2b + 2a = 0 \end{cases} \Rightarrow \begin{cases} a = 1 \\ b = 5 \\ c = 8 \end{cases}$

d'où $y_0 = x^2 + 5x + 8$ et $y = y_1 + y_0 = (\alpha + \beta x)e^x + x^2 + 5x + 8$ solutions de (E)

2/ (E) : $y'' - 2y' = x^2 + x$

(E') : $y'' - 2y' = 0$; $r^2 - 2r = 0$; $r_1 = 0$, $r_2 = 2$

$y_1 = \alpha e^{r_1 x} + \beta e^{r_2 x} = \alpha + \beta e^{2x}$ solutions de (E')

• $g(x) = x^2 + x$; $b = 0$; $a = -2 \neq 0$; $y_0 = ax^3 + bx^2 + cx + d$ solution particulière de (E)

$y_0' = 3ax^2 + 2bx + c$; $y_0'' = 6ax + 2b$

$y_0'' - 2y_0' = x^2 + x \Rightarrow \begin{cases} a = -1/6 \\ b = 1/2 \\ c = 1/2 \end{cases} \Rightarrow y_0 = -\frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{1}{2}x$

$y = y_1 + y_0 = \alpha + \beta e^{2x} - \frac{1}{6}x^3 + 1/2\, x^2 + 1/2\, x$ solutions de (E)

3/ $y'' = x^2 + x$; $y' = \frac{x^3}{3} + \frac{x^2}{2} + \alpha$; $y = \frac{x^3}{3} + \frac{x^2}{2} + \alpha x + \beta$

4/ (E): $y'' - 4y' + 3y = (x+1)e^{2x}$

• (E') : $y'' - 4y' + 3y = 0$; $r^2 - 4r + 3 = 0$; $\Delta = 4$, $r_1 = 1$, $r_2 = 3$

$y_1 = \alpha e^x + \beta e^{3x}$ solutions de (E')

• $g(x) = e^{mx} \cdot P(x)$ où $m = 2$ n'est pas solution de l'eq caractéristique donc la solution particulière est de la forme $y_0 = (ax + b)e^{2x}$

$y_0' = (a + 2b + 2ax)e^{2x}$; $y_0'' = (4a + 4b + 4ax)e^{2x}$

$y_0'' - 4y_0' + 3y_0 = (x+1)e^{2x} \Leftrightarrow 4a + 4b + 4ax - 4a - 8b - 8ax + 3ax + 3b = x + 1 \Rightarrow \begin{cases} a = -1 \\ b = -1 \end{cases}$

d'où $y_0 = (-x - 1)e^{2x}$ et $y = y_1 + y_0 = \alpha e^x + \beta e^{3x} - (x+1)e^{2x}$ solutions de (E)

5/ (E) : $y'' - 4y' + 3y = (x+1)e^x$

• (E') : $y'' - 4y' + 3y = 0$; $y_1 = \alpha e^x + \beta e^{3x}$ solutions

• $m = 1$ est solution de l'eq. caractéristique donc $y_0 = x(ax + b)e^x = (ax^2 + bx)e^x$

$y_0' = (2ax + b + ax^2 + bx)e^x$; $y_0'' = (2a + 2ax + b + 2ax + b + ax^2 + bx)e^x$

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• $y_0'' - 4y_0' + 3y_0 = (x+1)e^x \Rightarrow a = -1/4$ et $b = -3/4 \Rightarrow y_0 = (-\frac{1}{4}x^2 - \frac{3}{4}x)e^x$

et $y = y_1 + y_0 = \alpha e^x + \beta e^{3x} + (-\frac{1}{4}x^2 - \frac{3}{4}x)e^x$ solutions de (E)

6/ (E) : $y'' - 2y' + y = (x+1)e^x$

• (E') : $y'' - 2y' + y = 0$; $r^2 - 2r + 1 = 0$; $\Delta = 0$, $r = 1$; $y_1 = (\alpha + \beta x)e^x$

• m = 1 est solution double de l'eq caractéristique donc $y_0 = x^2(ax+b)e^x$

car $y_0 = (ax^3 + bx^2)e^x$

$y_0'' - 2y_0' + y_0 = (x+1)e^x \Rightarrow a = 1/6$; $b = 1/2$; $y_0 = (\frac{1}{6}x^3 + \frac{1}{2}x^2)e^x$

et $y = y_1 + y_0 = (\alpha + \beta x)e^x + (\frac{1}{6}x^3 + \frac{1}{2}x^2)e^x = (\frac{1}{6}x^3 + \frac{1}{2}x^2 + \beta x + \alpha)e^x$

7/ (E) : $y'' - 4y' + 3y = \sin 2x$

• (E') : $y'' - 4y' + 3y = 0$; $r^2 - 4r + 3 = 0$; $y_1 = \alpha e^x + \beta e^{3x}$

• La solution particulière y_0 de la forme $y_0 = a\sin 2x + b\cos 2x$

$y_0' = 2a\cos 2x - 2b\sin 2x$ $\quad y_0'' = -4a\sin 2x - 4b\cos 2x$

$y_0'' - 4y_0' + 3y_0 = \sin 2x \Rightarrow \begin{cases} -a + 8b = 1 \\ -b - 8a = 0 \end{cases} \Rightarrow \begin{cases} a = -1/65 \\ b = 8/65 \end{cases}$

d'où $y = y_1 + y_0 = \alpha e^x + \beta e^{3x} - \frac{1}{65}\sin 2x + \frac{8}{65}\cos 2x$ solutions de (E)

8/ (E) : $y'' - 4y' + 3y = e^x + \cos x$; (E') : $y'' - 4y' + 3y = 0$; $y_1 = \alpha e^x + \beta e^{3x}$

• La solution particulière y_0 de E est telle que : $y_0 = u_0 + v_0$ où u_0 et v_0 sont respectivement solution particulière de équations : $y'' - 4y' + 3y = e^x$ et $y'' - 4y' + 3y = \cos x$

$u_0 = x(ae^x) = axe^x$, $u_0' = (a + ax)e^x$, $u_0'' = (2a + ax)e^x$

$u_0'' - 4u_0' + 3u_0 = e^x \Rightarrow a = -1/2$. $\Rightarrow u_0 = -\frac{x}{2}e^x$

$v_0 = a\cos x + b\sin x$; $v_0' = -a\sin x + b\cos x$; $v_0'' = -a\cos x - b\sin x$

$v_0'' - 4v_0' + 3v_0 = \cos x \Rightarrow a = 3/22$ et $b = -2/11 \Rightarrow v_0 = \frac{3}{22}\cos x - \frac{2}{11}\sin x$

d'où $y_0 = -\frac{x}{2}e^x + \frac{3}{22}\cos x - \frac{2}{11}\sin x$ et $y = \alpha e^x + \beta e^{3x} - \frac{x}{2}e^x + \frac{3}{22}\cos x - \frac{2}{11}\sin x$ Sols de (E)

9/ (E) : $y'' - 4y' + 3y = \operatorname{ch} x = \frac{e^x}{2} + \frac{e^{-x}}{2}$; (E') : $y'' - 4y' + 3y = 0$, $y_1 = \alpha e^x + \beta e^{3x}$

• $y_0 = u_0 + v_0$ Sols particulière de (E) ; u_0 solution particulière de $y'' - 4y' + 3y = \frac{e^x}{2}$

et v_0 solution particulière de $y'' - 4y' + 3y = \frac{e^{-x}}{2}$

• $u_0 = axe^x$; $u_0' = (a + ax)e^x$, $u_0'' = (2a + 2ax)e^x$; $y_0'' - 4u_0' + 3u_0 = \frac{e^x}{2} \Rightarrow a = -1/4$

• $v_0 = ae^{-x}$; $v_0' = -ae^{-x}$; $v_0'' = ae^{-x}$; $v_0'' - 4v_0' + 3v_0 = \frac{e^{-x}}{2} \Rightarrow a = 1/16$

d'où $y_0 = -\frac{1}{4}xe^x + \frac{1}{16}e^{-x}$ et $y = \alpha e^x + \beta e^{3x} - \frac{x}{4}e^x + \frac{e^{-x}}{16}$ solutions de

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